

NAG Toolbox for MATLAB

f07pd

1 Purpose

f07pd computes the Bunch–Kaufman factorization of a real symmetric indefinite matrix, using packed storage.

2 Syntax

```
[ap, ipiv, info] = f07pd(uplo, n, ap)
```

3 Description

f07pd factorizes a real symmetric matrix A , using the Bunch–Kaufman diagonal pivoting method and packed storage. A is factorized as either $A = PUDU^T P^T$ if **uplo** = 'U' or $A = PLDL^T P^T$ if **uplo** = 'L', where P is a permutation matrix, U (or L) is a unit upper (or lower) triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 diagonal blocks; U (or L) has 2 by 2 unit diagonal blocks corresponding to the 2 by 2 blocks of D . Row and column interchanges are performed to ensure numerical stability while preserving symmetry.

This method is suitable for symmetric matrices which are not known to be positive-definite. If A is in fact positive-definite, no interchanges are performed and no 2 by 2 blocks occur in D .

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $PUDU^T P^T$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as $PLDL^T P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

A contains details of the block diagonal matrix D and the multipliers used to obtain the factor U or L as specified by **uplo**.

2: **ipiv**(*) – int32 array

Note: the dimension of the array **ipiv** must be at least $\max(1, n)$.

Details of the interchanges and the block structure of D . More precisely,

if **ipiv**(i) = $k > 0$, d_{ii} is a 1 by 1 pivot block and the i th row and column of A were interchanged with the k th row and column;

if **uplo** = 'U' and **ipiv**($i - 1$) = **ipiv**(i) = $-l < 0$, $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i - 1)$ th row and column of A were interchanged with the l th row and column;

if **uplo** = 'L' and **ipiv**(i) = **ipiv**($i + 1$) = $-m < 0$, $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i + 1)$ th row and column of A were interchanged with the m th row and column.

3: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **ipiv**, 5: **info**.

info > 0

If **info** = i , $d(i, i)$ is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

If **uplo** = 'U', the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon P|U||D||U^T|P^T,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If **uplo** = 'L', a similar statement holds for the computed factors L and D .

8 Further Comments

The elements of D overwrite the corresponding elements of A ; if D has 2 by 2 blocks, only the upper or lower triangle is stored, as specified by **uplo**.

The unit diagonal elements of U or L and the 2 by 2 unit diagonal blocks are not stored. The remaining elements of U or L overwrite elements in the corresponding columns of A , but additional row interchanges must be applied to recover U or L explicitly (this is seldom necessary). If **ipiv**(i) = i , for $i = 1, 2, \dots, n$ (as is the case when A is positive-definite), then U or L are stored explicitly in packed form (except for their unit diagonal elements which are equal to 1).

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to f07pd may be followed by calls to the functions:

f07pe to solve $AX = B$;

f07pg to estimate the condition number of A ;

f07pj to compute the inverse of A .

The complex analogues of this function are f07pr for Hermitian matrices and f07qr for symmetric matrices.

9 Example

```

uplo = 'L';
n = int32(4);
ap = [2.07;
      3.87;
      4.2;
      -1.15;
      -0.21;
      1.87;
      0.63;
      1.15;
      2.06;
      -1.81];
[apOut, ipiv, info] = f07pd(uplo, n, ap)

apOut =
    2.0700
    4.2000
    0.2230
    0.6537
    1.1500
    0.8115
   -0.5960
   -2.5907
    0.3031
    0.4074
ipiv =
        -3
        -3
         3
         4
info =
```

0